Chapter 10

Fix-flex Accounting in an Open Economy

Once one has estimated the price elasticities of import demand and export supply, as we have in the preceding chapter, the question naturally arises of how to integrate the resulting functional relationships into the overall model structure, if an overall model is what one's after. This chapter solves some not particularly deep but somewhat tricky accounting issues that arise thereby, namely when one attepmts to combine several demand-driven and several supply-constrained sectors in an open economy such as that of Ghana. This accounting would be at the heart of a dynamic model if building one could fit into this already ambitious project. As it is, it is something of a note on the margins, as it were.

10.1 A Fix-Flex Framework in an Open Economy

The problem we will solve here is the determination of imports and nontraded production given aggregate demand levels, when for some of the sectors nontraded production meets demand, but for other sectors total nontraded production is given in advance. Suppose that all relaive prices are known (in a model, they would evolve dynamically in a different module); thus in demand-constrained sectors the allocation of production between nontraded goods and exports is given. Likewise, assume we know the export volumes for the sectors whose nontraded production is demand-driven.

Now consider an increase in aggregate demand, and the way it gets distributed among imports and nontraded goods. In demand-driven sectors, domestic production will grow along with imports, maintaining a fixed import-to-nontraded volume ratio. This could be the case for manufactures in Ghana. On the other hand, for supply-constrained sectors, such as agriculture, all extra demand will spill directly into imports.

So far, so simple. However, the picture is complicated by two extra developments: fistly, demand-driven and supply-constrained sectors use each other's output as intermediate inputs. Secondly, domestic trasport and retail services, although demand driven, are only supplied by domestic producers, and are not directly demanded by consumers, but rather are bundled with the final goods that consumers demand.

While all these restrictions could conceivably be formulated as implicit constraints on the solution, we feel it is desirable to make the computations explicit. That will make them more transparent, clarify the causal structrure, and speed up the numerical solution process. As several hundred runs, at least, are typically necessary for numerical estimation and sensitivity analysis of a dynamic model of the size ours would have to be, speed is not a negligible factor.

Let us formulate the problem precisely. First, consider the flow equilibrium condition in the goods markets, in real terms - it is always observed as any product or service bought by somebody must be sold by somebody else. Let E be the vector of exports by sector, A the total absorbtion (private consumption plus investment plus government final demand), II the total intermediate input demand, I imports, and N non traded production. All of these are vectors with the elements corresponding to sectors. Further, let S denote the square matrix describing derived demand for transport and retail, so that derived demand for transport and retail equals $S(I+N)$. All rows of S are zero except the row corresponding to transport and retail services, with the entries in this row containing the sector-specific values of the retail markup. As transport and retail itself does not attract a retail markup (it can use retail as intermediate input, but that is taken care of in the Leontief matrix below), the diagonal entries of S are all zero, and thus $S^2 = 0$.

With the notation just introduced, we can formulate the flow equilibrium condition in the goods markets, in real terms:

$$E + A + II + S(I + N) = I + N + E \tag{10.1}$$

Now intermediate inputs can be decomposed in intermediate inputs used by private sector for export production, by private sector for nontraded production, and by the government. We assume that the private sector uses a Leontief technology, while the government determines the composition of its intermediate input demand arbitrarily. Thus we have

$$II = II_g + LE + LN \tag{10.2}$$

with L being the Leontief matrix. For easier manipulation let us define total demand as

$$D = I + N \tag{10.3}$$

Then (10.1), after canceling out the exports on both sides and inserting (10.2) and (10.3), becomes

$$\begin{aligned} D &= (A + II_g + LE) + LN + SD \Leftrightarrow & (10.4)\\ (1 - S) &= D\,(A + II_g + LE) + LN \Leftrightarrow & (10.5)\\ D &= (1 + S)\,[(A + II_g + LE) + LN] & (10.6) \end{aligned}$$

The second transition holds as $S^2 = 0$, and therefore $(1 - S)^{-1} = (1 + S)$ (with 1 denoting the unity matrix throughout). The equation (10.6) is actually a quite intuitive version of the product flow equilibrium: total demand equals absorption plus intermediate input demand, with the retail/transport markup on top of everything.

If we denote

$$\begin{aligned} \tilde{A} &= (1 + S)\,(A + II_g + LE) & (10.7)\\ \tilde{L} &= (1 + S)L & (10.8) \end{aligned}$$

then (10.6) can be re-written as

$$D = \tilde{A} + \tilde{L}N \tag{10.9}$$

Here $\tilde{A}$ is a combination of known quantities and is therefore known; likewise $\tilde{L}$. Having thus converted the basic conservation law into a form suitable for convenient manipulation, let us now turn to allocating demand between imports and nontraded goods.

To begin with, let's pretend all sectors are demand driven and therefore for each sector i the ratio σ_i of nontraded goods to imports is a function of relative price and therefore fixed for our purposes. Let σ be a diagonal matrix whose diagonal entries are σ_i. Then the constant ratio relationship can be written as

$$N = \sigma I + SD \tag{10.10}$$

because the domestic transport and retail services are not subject to that ratio relationnship, but are supplied purely domestically, as a markup on demand.

Using (10.9) we can re-write (10.10) as

$$\begin{aligned} N &= \sigma(D - N) + SD = & (10.11)\\ &= \sigma(\tilde{A} + \tilde{L}N - N) + S(\tilde{A} + \tilde{L}N) = & (10.12)\\ &= (\sigma + S)\tilde{A} + [(\sigma + S)\tilde{L} - \sigma)]N \qquad \Leftrightarrow & (10.13)\\ \left[1 + \sigma - (\sigma + S)\tilde{L}\right] N &= (\sigma + S)\tilde{A} & (10.14) \end{aligned}$$

We are now almost at our goal. However, the reader must at this point resist the urge to invert the matrix in front of N in (10.14), as (10.14) is not in truth valid for all sectors, but only for demand-driven ones. All of our manipulations starting with (10.10) only consisted in rearranging terms without mixing up the different sectors, and thus the equality (10.14) is still true for the demand-driven sectors, but not for supply - constrained ones.

Let us denote

$$\begin{aligned} B &= 1 + \sigma - (\sigma + S)\tilde{L} & (10.15)\\ &= 1 + \sigma - (\sigma + S)(1 + S)L & (10.16)\\ A^{(1)} &= (\sigma + S)\tilde{A} & (10.17)\\ &= [(\sigma + S)(1 + S)](A + II_g + LE) & (10.18) \end{aligned}$$

Then (10.14) can be rewritten as

$$BN = A^{(1)} \tag{10.19}$$

Note that the matrix B is constant and is computed in a straigthforward manner from the known matrices describing import to nontraded ratio, retail markups and the Leontief technology. Likewise, the vector $A^{(1)}$ is easily computed by multiplying the exogenous demand injections by a constant matrix.

Now (10.19) is still only true for the elements corresponding to demand-driven sectors. To isolate these, let us split the vector N into the supply-constrained and demand-driven components

$$N = \begin{pmatrix} N_s \\ N_d \end{pmatrix} \tag{10.20}$$

and look at the corresponding block-matrix representation of (10.19):

$$\begin{aligned} \begin{pmatrix} B_{ss} & B_{sd} \\ B_{ds} & B_{dd} \end{pmatrix} \begin{pmatrix} N_s \\ N_d \end{pmatrix} &= \begin{pmatrix} A_s^{(1)} \\ A_d^{(1)} \end{pmatrix} \Leftrightarrow \\ \begin{pmatrix} B_{ss}N_s + B_{sd}N_d \\ B_{ds}N_s + B_{dd}N_d \end{pmatrix} &= \begin{pmatrix} A_s^{(1)} \\ A_d^{(1)} \end{pmatrix} & (10.21) \end{aligned}$$

As we have said before, only the lower half of (10.21), that is the part referring to demand-driven sectors is true. Thus we finally arrive at the precisely true expression for the demand-driven sectors:

$$B_{ds}N_s + B_{dd}N_d = A_d^{(1)} \tag{10.22}$$

As the nontraded output of the supply-constrained sectors vector N_s is known, we can solve (10.22) for N_d as

$$N_d = B_{dd}^{-1}\left(A_d^{(1)} - B_{ds}N_s\right) \tag{10.23}$$

Inserting that into (10.20) we obtain the whole nontraded output vector N. Inserting it into (10.9) gives us the total demand D, and solving (10.3) for I gives us imports by sector.

10.2 Summary

This section derived the accounting for determining total volumes of imports and nontraded production given total absorbtion and export volumes, in an open economy combining supply-constrained and demand-driven sectors. It took into account relative price-determined ratios between imports and nontraded goods, retail ratios, and the demand for intermediate inputs.

Once the derivation is done, the actual computation required is quite simple. This can be a useful component of a dynamic model of a fix-flex open economy with an arbitary number of sectors.